

FROSTI runs at about 75 °C (348 K). This short note is to show, with numbers, why photon bunching from a source this cool cannot be detected, and why neither longer runs nor better optics can change that. In every step I use the most favorable value physics allows, so the result is a hard limit, not a guess.

Claim 1: The source temperature caps the signal

Bunching happens because thermal photons like to arrive in groups. How strong this grouping is depends on one number, the average number of photons per mode:

$$\bar{n} = \frac{1}{e^{hc/\lambda kT} - 1}, \quad \frac{hc}{k} = 14\,388 \text{ } \mu\text{m K}. \quad (1)$$

The photon count per mode fluctuates with variance $\bar{n}(1 + \bar{n})$. The first part is normal shot noise, which is random and different at each detector. The extra part is the bunching. So the bunching signal can never be larger than a fraction $\bar{n}$ of the shot noise. This is a hard ceiling set by the source alone. No detector, lens, or amount of patience can raise it.

Our detectors see 2.7 to 5.0 μm . Taking the best wavelength in that range (5.0 μm):

Source	T [K]	$\bar{n}$
FROSTI	348	2.6×10^{-4}
Globalar	1400	0.16
Neben thesis lamp (at 1.55 μm)	2540	0.027

The gap is huge because $\bar{n}$ falls off exponentially as the source gets cooler (Figure 1). Going from a globalar down to FROSTI costs a factor of about 600.

The real detector makes things worse. Only the photon part of our noise carries the bunching, so the correlation we can actually measure is

$$C_{\text{cor}} = \bar{n} \times \frac{S_{\text{shot}}}{S_{\text{shot}} + S_{\text{amp}}}, \quad (2)$$

where $S_{\text{shot}} = 2h\nu P$ is the photon shot noise at detected power P and $S_{\text{amp}} = \text{NEP}^2$ is the detector's own noise (PDAVJ5 data sheet: $\text{NEP} = 14 \text{ pW}/\sqrt{\text{Hz}}$). Our DC readings (252 and 179 mV at a conversion gain of $2 \times 10^5 \text{ V/W}$) mean we detect only about 1 μW of light. At that power the fraction in the equation is about 5×10^{-4} , so today

$$C_{\text{cor}} \approx 2.6 \times 10^{-4} \times 5 \times 10^{-4} \approx 1 \times 10^{-7}. \quad (3)$$

Our measurement reaches about 10^{-4} . The signal sits a thousand times below our floor. The null result we got is exactly what these numbers predict.

Claim 2: Longer runs cannot close the gap

The signal to noise ratio grows only with the square root of time:

$$\text{SNR} = C_{\text{cor}} \sqrt{2 \Delta f t}. \quad (4)$$

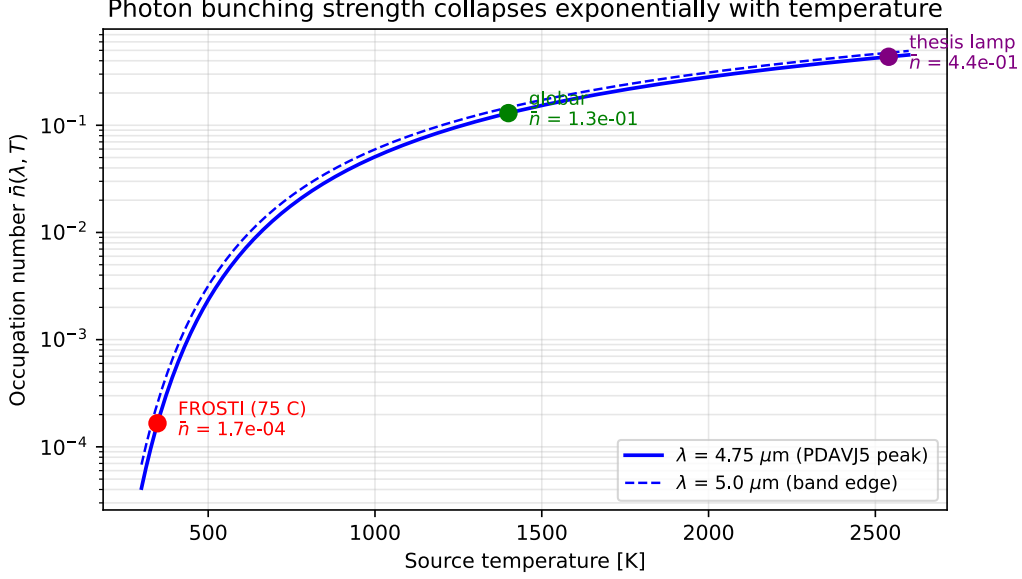


Figure 1: Bunching strength $\bar{n}$ against source temperature. The vertical axis is logarithmic. FROSTI sits nearly three orders of magnitude below a globar.

With a generous clean bandwidth of $\Delta f = 10 \text{ kHz}$, reaching $C_{\text{cor}} \approx 10^{-7}$ at $\text{SNR} = 5$ takes

$$t = \frac{25}{2 \times 10^4 \times (10^{-7})^2} \approx 10^{11} \text{ s} \approx \text{thousands of years.} \quad (5)$$

Square root scaling means every factor of 10 in sensitivity costs a factor of 100 in time. A gap of 1000 costs a factor of a million. This is not a scheduling problem. It is a physics problem.

Claim 3: Better optics cannot close the gap either

More light does help, since the fraction in Claim 1 grows with P . But there is a firm limit on how much light any optical setup can collect, and it comes from a theorem. Conservation of étendue (the brightness theorem) says that no passive optical system, no matter how large, can make an image brighter than the source itself. So the most power any setup can put on our detector is the source's in-band radiance times the largest light-gathering our detector can accept, which is πA for a detector of area A .

For a perfect blackbody at 348 K, only 3.2% of the emitted power falls below $5 \mu\text{m}$, giving 27 W/m^2 in our band. Times our $1 \times 1 \text{ mm}^2$ detector area:

$$\boxed{P_{\text{max}} \approx 27 \mu\text{W}.} \quad (6)$$

This assumes perfect emissivity, lossless optics, and light arriving from a full half sphere, all at once. Any real setup gets less. Note that $27 \mu\text{W}$ is well below the detector's $200 \mu\text{W}$ saturation, so the limit here is thermodynamics, not our hardware. Putting P_{max} into the equations of Claim 1 gives $C_{\text{cor}} \approx 1.9 \times 10^{-6}$, which needs about 11 years of data. Even if the detector could somehow be fed its full $200 \mu\text{W}$, which the theorem forbids for this source, the wait is still about 85 days.

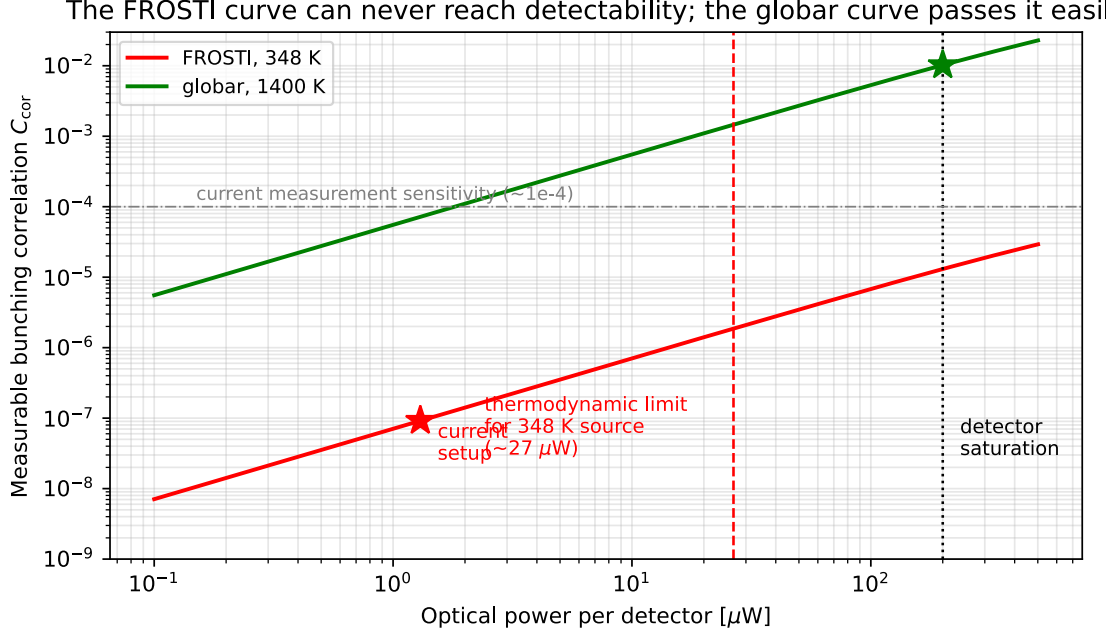


Figure 2: Measurable correlation against detected power. The FROSTI curve hits the thermodynamic wall at 27 μW , far below our sensitivity line. The global curve crosses the line easily.

The same setup on a hot source works in seconds

A global (the glowing SiC rod inside every IR spectrometer, about 1400 K, an inexpensive catalog part) changes only T in the equations above. Its $\bar{n}$ is about 0.15, and it is bright enough that reaching 200 μW on the detector is easy; we would turn the gain down. Then $C_{\text{cor}} \approx 10^{-2}$ and the detection takes about 12 seconds with the same beamsplitter, detectors, digitizer, and code.

Configuration	C_{cor}	Time to SNR = 5
Current setup (1.3 μW)	$\sim 1 \times 10^{-7}$	$\sim 4,700$ years
Étendue limit, 27 μW	1.9×10^{-6}	~ 11 years
Detector saturation (not reachable)	1.3×10^{-5}	~ 85 days
Global 1400 K at 200 μW	$\sim 1 \times 10^{-2}$	~ 12 s

Every FROSTI row above grants more than physics allows, and each still falls short by orders of magnitude.

Normalized Cross-Correlation at Zero delay:

$$C = \frac{1}{N \sigma_x \sigma_y} \sum_{i=1}^N x_i x_i$$

the two timestreams
standard deviation

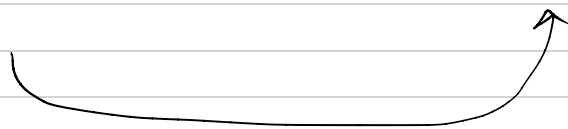
if x_i and y_i related $C(c) = 1$

if x_i and y_i unrelated $C(c) \neq 1 \rightarrow$ random number

how big? size of noise floor

because they are random independent variables:

$$S \equiv \sum_{i=1}^N x_i x_i \xrightarrow[\text{variance}]{\text{take}} \text{Var}(S) = N \sigma_x^2 \sigma_y^2$$



$$\langle (x_i y_i)^2 \rangle = \langle x_i^2 \rangle \langle y_i^2 \rangle = \sigma_x^2 \sigma_y^2$$

$$S_{\text{RMS}} = \sqrt{\text{Var}(S)} = \sqrt{N \sigma_x^2 \sigma_y^2} = \sqrt{N} \sigma_x \sigma_y$$

insert in C :

random walk

$$C = \frac{S_{\text{RMS}}}{N \sigma_x \sigma_y} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

$$C_{RMS} = \frac{1}{\sqrt{N_{ind}}} \quad \text{thesis eq (9)}$$

↘ number of independent sample

how fast can signal of bandwidth Δf change? Nyquist-Shannon

$$N_{ind} = \frac{t}{\text{sample speed}} = \frac{t}{1/2\Delta f} = 2\Delta f t$$

or, a band limited signal produce one sample every $\frac{1}{2\Delta f}$ second (Nyquist rate for that bandwidth)

$$\rightarrow N_{ind} = 2\Delta f t \text{ in time } t$$

insert in C_{RMS} (thesis eq 9)

$$C_{RMS} = \frac{1}{\sqrt{N_{ind}}} = \frac{1}{\sqrt{2\Delta f t}}$$

C_{RMS} is noise and C_{cor} is signal, so signal-to-noise ratio is

$$SNR = \frac{C_{cor}}{C_{RMS}} = \frac{C_{cor}}{\frac{1}{\sqrt{2\Delta f t}}} = C_{cor} \sqrt{2\Delta f t}$$

$$SNR = C_{cor} \sqrt{2\Delta f t} \rightarrow t = \frac{SNR^2}{C_{cor}^2 2\Delta f}$$

$$t = \frac{SNR^2}{C_{cor}^2 2 \Delta F}$$

$SNR = 5$, $\Delta F = 10 \text{ kHz}$ the widest clean band

from $8 \text{ kHz} - 20 \text{ kHz}$

↙ cumb tail dies ↘ electronic bump

$$t = \frac{5^2}{C_{cor}^2 2 (10)^4} = \frac{1.25 \times 10^{-3}}{C_{cor}^2} \text{ second}$$

$$C_{cor} = \bar{n} \times \frac{S_{shot}}{S_{shot} + S_{amp}} \rightarrow \text{check appendix A}$$

$$S_{amp} = NEP^2, \left[\text{power spectral density} = (\text{amp spectral density})^2 \right]$$

for PDA VJ5 $\rightarrow NEP = 14 \frac{\text{pW}}{\sqrt{\text{Hz}}} = 1.4 \times 10^{-11} \frac{\text{W}}{\sqrt{\text{Hz}}}$

↙ amplitude spectral density

$$S_{amp} = 2 \times 10^{-22} \frac{\text{W}^2}{\text{Hz}}$$

$S_{shot} = 2 h \nu P$ the one-sided power spectral density of power flux

photon arrives at rate $R = \frac{P}{h \nu}$ $\rightarrow$ beam average power $\rightarrow$ each photon energy

$$h\nu = \frac{hc}{\lambda} \quad \left\{ \begin{array}{l} h = 6.63 \times 10^{-34} \\ c = 3.0 \times 10^8 \text{ m/s} \\ \lambda = 4.75 \times 10^{-6} \text{ m (PDAVJ 5's peak wavelength is } 4.75 \mu\text{m)} \end{array} \right.$$

$$h\nu = \frac{1.99 \times 10^{-25}}{4.75 \times 10^{-6}} = 4.2 \times 10^{-20} \text{ J}$$

$$S_{\text{shot}} = 2h\nu P = 8.4 \times 10^{-20} P \quad (P \text{ in watts})$$

$$\text{Froshi} \rightarrow P = 1.3 \mu\text{W} = 1.3 \times 10^{-6} \text{ W}$$

$$\bar{n} = 2.6 \times 10^{-4} \quad (T = 348 \text{ K})$$

$$\text{etendue} \rightarrow p = 27 \mu\text{W} = 27 \times 10^{-6} \text{ W} \quad (\text{the maximum any optic can deliver from a } 348 \text{ K source})$$

$$\bar{n} = 2.6 \times 10^{-4} \quad (T = 348 \text{ K})$$

$$\text{detector} \quad P = 200 \mu\text{W} = 200 \times 10^{-6} \text{ W}$$

$$\bar{n} = 2.6 \times 10^{-4} \quad (T = 348 \text{ K})$$

$$\text{Global} \quad P = 200 \mu\text{W} = 200 \times 10^{-6} \text{ W}$$

$$\bar{n} = 0.13 \quad (T = 1400 \text{ K})$$

Appendix A

eq 7 thesis

reflection-transmission covariance

$$C_{cor} = \frac{\sigma_{rt}^2}{\sqrt{(\sigma_r^2 + \sigma_{amp}^2)(\sigma_t^2 + \sigma_{amp}^2)}}$$

$$\sigma_{rt}^2 \approx (R_s T_s + R_p T_p) \approx \bar{n}^2 \quad (\text{there is no shot term } \delta)$$

$$\sigma_{rt}^2 = S_{\text{bunch}}$$

$$\sigma_t^2 = S_{\text{bunch}}^{(t)} + S_{\text{shot}}^{(t)}$$

eq 5 for transmission

$$\sigma_r^2 = S_{\text{bunch}}^{(r)} + S_{\text{shot}}^{(r)}$$

every σ^2 is power spectral density of power fluctuation in units of (W^2/Hz)

$$\sigma_{rt}^2 = \int_{\nu_1}^{\nu_2} d\nu (h\nu)^2 \frac{A \Omega \nu^2}{c^2} \underbrace{(R_s T_s + R_p T_p)}_{\text{shared}} \frac{1 e^2}{(e^{h\nu/kT} - 1)^2} \bar{n}^2(\nu)$$

$$\sigma_t^2 = \int_{\nu_1}^{\nu_2} d\nu (h\nu)^2 \frac{A \Omega \nu^2}{c^2} \left((T_s^2 + T_p^2) \frac{e^2}{(e^{h\nu/kT} - 1)^2} + (T_s + T_p) \frac{e}{(e^{h\nu/kT} - 1)} \right)$$

shot noise $\propto (\bar{n}(\nu))$

$$K = (h\nu)^2 \frac{A \Omega \nu^2}{c^2} \Delta\nu \quad \text{for small frequency } K \text{ is constant}$$

Units Cancell each other

→ coming from integral

why $\bar{n}^2 \propto$ bunching and $\bar{n} \propto$ shot noise?

remember the building that I was saying? even if people are coming out independtly, the number of people coming out of bulding grows like n
the variance is $\bar{n}$ (this is for any mode)

now bunching = pairing fluctuation. here photons are not independent. thermal photons like to travel together. bunching noise isn't about how many photons (people) there are, it's about how many pairs travel together.

the number of possible pairs among $\bar{n}$ photons grows like $\bar{n}^2$.

shot nois $\propto \bar{n} \rightarrow$ one photon is enough to be counted $\rightarrow$ uncorrelated

bunching $\propto \bar{n}^2 \rightarrow$ need two photons to make a pair $\rightarrow$ correlated, shared

$$\text{shot noise} \times \bar{n} = \text{bunching} \rightarrow S_{\text{shot}} \bar{n} = S_{\text{bunching}}$$

now you may ask why $C_{\text{cor}} \propto \bar{n}$? but it's NOT!

$$C_{\text{cor}} = \frac{S_{\text{bunch}}}{\sqrt{(S_{\text{bunch}} + S_{\text{shot}} + S_{\text{amp}})(S_{\text{bunch}} + S_{\text{shot}} + S_{\text{amp}})}}$$

$$C_{\text{Cor}} = \frac{S_{\text{bunch}}}{S_{\text{bunch}} + S_{\text{shot}} + S_{\text{amp}}} \approx \frac{S_{\text{bunching}}}{S_{\text{shot}} + S_{\text{amp}}} = \frac{\bar{n} S_{\text{shot}}}{S_{\text{shot}} + S_{\text{amp}}}$$

$\swarrow$
very small

$$C_{\text{Cor}} \approx \bar{n} \frac{S_{\text{shot}}}{S_{\text{shot}} + S_{\text{amp}}} \quad \checkmark \quad \text{😊}$$

$\frac{W^2/\text{Hz}}{W^2/\text{Hz}}$

Correlation Coefficient

at what T we can detect photon bunching?

$$h\nu = \frac{hc}{\lambda} = 4.19 \times 10^{-20} \text{ J (for our detector } \lambda = 4.75 \times 10^{-6} \text{)}$$

$$S_{\text{amp}} = NEP^2 = 1.96 \times 10^{-22} \text{ W}^2/\text{Hz (for our detector)}$$

$$\text{SNR} = 5, \Delta f = 10^4 \text{ Hz}$$

$$T = 700 \text{ K} \rightarrow \frac{hc}{\lambda kT} = 8.71 \times \frac{348}{700} = 4.33 \quad / \quad k = 1.381 \times 10^{-23}$$

$$\bar{n} = \frac{1}{e^{4.33} - 1} = \frac{1}{76 - 1} = 1.33 \times 10^{-2}$$

$$P(700) = P(348) \times \frac{\int_{2.7}^5 \text{plank}(700) dA}{\int_{2.7}^5 \text{plank}(348) dA} = 1.06 \mu\text{W} \times 168 = 178 \mu\text{W}$$

$\underbrace{\hspace{15em}}_{\text{numerically}} \uparrow$

$$S_{\text{shot}} = 2h\nu P = 2(4.19 \times 10^{-20})(1.78 \times 10^{-4}) = 1.49 \times 10^{-23}$$

$$\frac{S_{\text{shot}}}{S_{\text{shot}} + S_{\text{amp}}} = 7.06 \times 10^{-2} \rightarrow C_{\text{Cor}} = (1.33 \times 10^{-2})(7.06 \times 10^{-2}) = 9.4 \times 10^{-4}$$

$$t = \frac{1.25 \times 10^{-3}}{C_{\text{cor}}^2} = 1420 \text{ s} \approx 23 \text{ min}$$

with current setup:

Temperature	$\bar{n}$	detector power	time to detect
348 K (75°C)	1.7×10^{-4}	1 μW	~ 7000 years
500 K (227°C)	2.3×10^{-3}	22 μW	~ 32 days
600 K (327°C)	6.5×10^{-3}	73 μW	~ 9 hours
700 K (427°C)	1.3×10^{-2}	178 μW	~ 23 min
800 K (527°C)	2.3×10^{-2}	354 μW	~ 2 mins